Exercise 24

Use a linear approximation (or differentials) to estimate the given number.

$$1/4.002$$

Solution

Compute the derivative of $y = 1/x$.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(\frac{1}{x}\right) \\ &= -\frac{1}{x^2}\end{aligned}$$

Consequently, the differential of $y = 1/x$ is

$$dy = -\frac{1}{x^2}\,dx.$$

In order to estimate $1/4.002$, set $x = 4$ and $dx = 0.002$.

$$dy = -\frac{1}{4^2}(0.002) = -0.000125$$

Note that dy here is the vertical distance from the function's actual value at $x = 4$ to the linear approximation's value at $x = 4.002$.

$$\frac{1}{4.002} \approx \frac{1}{4} + (-0.000125) = 0.249875$$